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Slow drying simulation in thick layers of granular products

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Abstract—The drying rate equation used in simulation models for low temperature drying which takes the air moisture effects as a humidity potential calculated from adiabatic saturation is valid. We propose to transform the partial differential equations model in dimensionless form with reference parameters in agreement with physical problems. Slow drying allows some simplifications and thus comes an analytical solution which depends on drying rate. One shows how to obtain the characteristics of the drying and wetting fronts.

1. INTRODUCTION

SLOW DRYING, i.e. at low temperature (air temperature of ambient to ambient plus 10°C) of thick layers of granular products is quite important in the agricultural sector (cereals, fodder, fruits,...). It is a low cost process, since the ambient air drying power is used with the possibility of an auxiliary energy source such as solar heat or fan power heat. The ambient air drying is adapted to biological products of low moisture content, i.e. without risk of bacterial deterioration; otherwise auxiliary heat is needed to rapidly evaporate some moisture until the product is stabilized.

As a particular example of the product, we took wood chips obtained by grinding forest waste. The granular nature of the material is ideal for automatic feeding of large capacity boilers. The period between grinding (in the spring) and burning (the following winter) is long and thus affords the possibility of drying at ambient air temperature. Daytime ventilation avoids any heating caused by bacterial growth.

The material used for simulation in the laboratory is expanded clay pellets used in civil engineering. The clay pellets can absorb up to 80% of water mass without any risk of degradation. We also take as an example the cooling of cereal grains after harvest or the dryeration process in drying maize.

We will study the case of the static drier where the product is stocked in a deep fixed bed (6 m for wood chips), ventilated by uniform airflow and hence considered as a one-dimensional problem. The optimization of an ambient air drier is more difficult to establish than a high temperature one, since the input data are meteorological.

There is much research literature into the simulation of low temperature grain drying [1, 2]. A complete simulation model comprising a partial differential equation (p.d.e.) for mass balance, heat balance, heat transfer and drying rate, can be solved with numerical techniques and some computer time [3, 4]. The results show a strong dependence of the product on the thin layer equation which is the key to the problem. Few models have been validated under true deep bed, low temperature, low airflow conditions. These features are expressed with a number of units of transfer [8] (proportional to length and inversely to mass rate) of the drier and is very important.

A particular class of model is the equilibrium model; there is a simplification of the p.d.e. model where the main assumption made is that equilibrium conditions exist between the drying air and the grain. The latter equation of the thin layer drying rate is substituted for an expression of the equilibrium moisture content of the grain. The resolution of the model is easier and shows the propagation of a drying front preceded by a wetting front (which experiments confirm) [5, 6].

Another simplified model that is frequently used is the logarithmic model of Hukill [7]. Assuming a special form of boundary conditions, it proposes an analytical solution of deep bed drying.

In this paper, we give a simplified resolution of the p.d.e. model for the slow drying of a deep bed for constant data input. This resolution leads to an examination of the validity of preceding simplified models and the more general solution of continuous drying in Keey [8].

We have carried out some experiments in the laboratory of slow drying of wood chips and expanded clay pellets which demonstrate drying front displacement without distortion (Fig. 1). This phenomenon can be observed in the case of corn cooling (Fig. 2). After a simple calculation based on enthalpy and mass balance in a stationary system fastened to the drying front, the author [9] of the last experiment gave the celerity.

2. DRYING MODEL (p.d.e.)

The following principal assumptions are made in the development of the grain drying model:

(a) the volume shrinkage is negligible during the drying process;

NOMENCLATURE

C massic heat [J kg^{-1} K^{-1}]
d particle diameter
L latent heat [J kg^{-1}]
M moisture content of the product (dry basis)
T temperature [°C]
t time
V flow rate [m s^{-1}]
W absolute humidity of air
x coordinate.

Greek symbols
α heat exchange coefficient [W m^{-2} K^{-1}]
β mass exchange coefficient [m s^{-1}]
ε bed porosity
ξ surface area per unit volume [m^2 m^{-3}]
ρ volumetric mass [kg m^{-3}].

Subscripts
a dry air
e equilibrium
g grain
i initial
o inlet
s saturation
v vapour
w water
wb wet bulb.

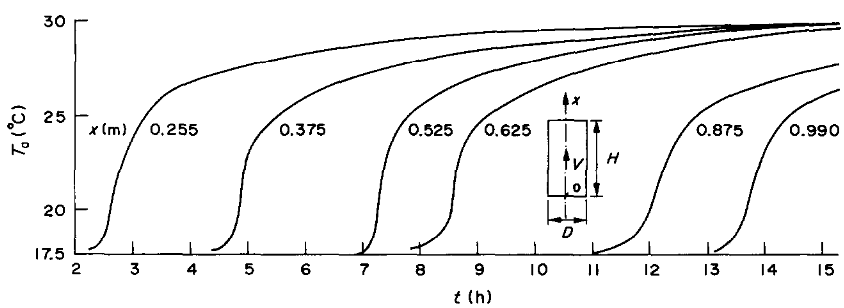

FIG. 1. Low drying of expanded clay pellets in thick layer. Air temperature curves: $T_{ao} = 30°C$; $T_s = 17.5°C$; $V = 0.67$ m s^{-1}; $M_i = 0.42$; cylindrical bin $H = 2$ m, $D = 0.5$ m.

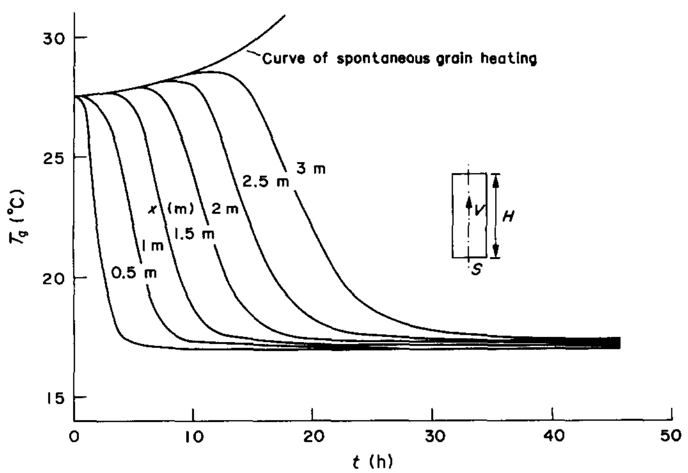

FIG. 2. Corn cooling with conditioned air [9], grain temperature curves: $T_{gi} = 27.5°C$; $T_{ao} = 17.4°C$; $V = 0.028$ m s^{-1}; $M_i = 0.25$; $W_o/W_s(T_{ao}) = 0.68$; cylindrical bin $H = 3$ m, $S = 1$ m^2.

(b) the conductive and radiative exchanges are negligible compared to convective exchange;

(c) the static bed is a cross-flow unidirectional system, with no transfer in transverse directions, and the flow rate is constant;

(d) the granular porous medium is described by two phase macroscopic functions T_a, T_g, W, M;

(e) the variation of volumetric mass of dry air ρ_a is negligible.

The theoretical analysis of drying of a static bed leads to a set of partial differential equations (Appendix).

Consider the following equations:

the mass balance for moist air

$$\rho_a\left(\varepsilon\frac{\partial W}{\partial t}+V\frac{\partial W}{\partial x}\right)+(1-\varepsilon)\rho_g\frac{\partial M}{\partial t}=0; \quad (1)$$

the enthalpy balance for moist air and grain

$$\rho_a(C_a+WC_v)\left(\varepsilon\frac{\partial T_a}{\partial t}+V\frac{\partial T_a}{\partial x}\right)=\xi\alpha(T_g-T_a)$$

$$-(1-\varepsilon)\rho_g C_v(T_g-T_a)\frac{\partial M}{\partial t} \quad (2)$$

$$(1-\varepsilon)\rho_g(C_g+MC_w)\frac{\partial T_g}{\partial t}$$

$$=\xi\alpha(T_a-T_g)+L\rho_g(1-\varepsilon)\frac{\partial M}{\partial t}; \quad (3)$$

the global enthalpy balance for moist air and grain

$$\rho_a(C_a+C_vW)\left(\varepsilon\frac{\partial T_a}{\partial t}+V\frac{\partial T_a}{\partial x}\right)$$

$$+(1-\varepsilon)\rho_g(C_g+MC_w)\frac{\partial T_g}{\partial t}=L^*(1-\varepsilon)\rho_g\frac{\partial M}{\partial t} \quad (4)$$

where

$$L^*=L^\circ+C_vT_a-C_wT_g+h_s.$$

Here L^* is the heat of vaporization which comprises sorption heat h_s. As this is generally negligible except in the final stages of drying and as its formulation is not simple, we will not refer to it further.

3. DRYING RATE EQUATION

Results from the thin layer drying experiments are used to construct an empirical equation where the rate of moisture loss is expressed as a function of the temperature, humidity and velocity of air

$$\frac{\partial M}{\partial t}=F(M,W,T_a).$$

Using this equation for describing the moisture exchange of the grain with air in a deep bed is even more justifiable when the temperature is low. Indeed thin layer (~ 1 cm) drying occurs when air characteristics are constant, since thick layer (~ 1 m) drying processes occur when air temperature changes over time which can induce temperature gradients in the grain.

This drying rate equation becomes a characteristic drying curve normalized as ($\dot{M}=\partial M/\partial t$) [10]

$$\frac{\dot{M}}{\dot{M}_i}=f\left(\frac{M-M_e}{M_i-M_e}\right) \quad (5)$$

where M_i is the initial grain moisture and $M_e = M_e(W, T_a)$ the equilibrium grain moisture.

Few products present a true, first period of constant drying rate (PCDR). In this case it is generally assumed that the grain surface remains saturated at a surface temperature equal to wet-bulb temperature. It is difficult to carry out experiments where a PCDR can be clearly observed. There are two reasons for this.

(1) When the initial temperature of the product is different to the wet-bulb temperature, one observes a transient phase which can hide the PCDR, and the critical point at the end of this period is expected near the maximum of the rate (p. 155 of ref. [8]).

(2) Accurate determination of the drying rate demands a continual acquisition by a computer which can display a statistical curve and its derivation. "It is sometimes stated that a PCDR can only be found when the accuracy of measurement is low" (p. 223 of ref. [10]). For example for foods, a PCDR can be noted only when the drying potential of air is very low or the moisture content of the product very high [11]. If one assumes that with or without PCDR, the maximum drying rate $\dot{M}_i$ corresponds to the mass transfer through the boundary layer on a wet surface, we can write (p. 123 of ref. [8])

$$(1-\varepsilon)\rho_g\dot{M}_i=-\rho_a\beta\xi(W_{wb}(T_g)-W) \quad (6)$$

$$L\rho_g(1-\varepsilon)\dot{M}_i=-\xi\alpha(T_a-T_g). \quad (7)$$

The Chilton–Colburn analogy allows equations (6) and (7) to be combined giving

$$L(W_{wb}-W)=(C_a+C_vW)(T_a-T_g)$$

$$W_{wb}=W_s(T_g)$$

which is the definition of the wet-bulb temperature and the adiabatic path on an air moisture chart.

If one examines the hypothesis behind this result, it is doubtful that the surface temperature of a drying product will be always the wet-bulb temperature. Indeed some factors must be examined (p. 235 of ref. [10]).

(1) The driving force for vapour diffusion through the boundary layer is molar concentration, or volumetric mass, or partial pressure (Fick's law). Taking the mass vapour–dry air ratio one introduces a corrective factor which increases with air temperature.

(2) The surface of the porous capillary medium exposed to the drying air is rarely completely wet. This fact influences the drying rate if the dimensions of the dry and wet patches are of the same order as the boundary layer thickness.

(3) The thermal equivalence between convective heat and mass transfer is justified for a sufficient air velocity for which conductive and radiative exchanges are negligible.

The point of the initial state of the grain on the Grosvenor humidity chart is determined (Fig. 3) by the intersection of the saturation humidity curve $W = W_s(T)$ and the line

$$L\rho_a\beta(W_s(T_g)-W)=\alpha(T_a-T_g) \quad (8)$$

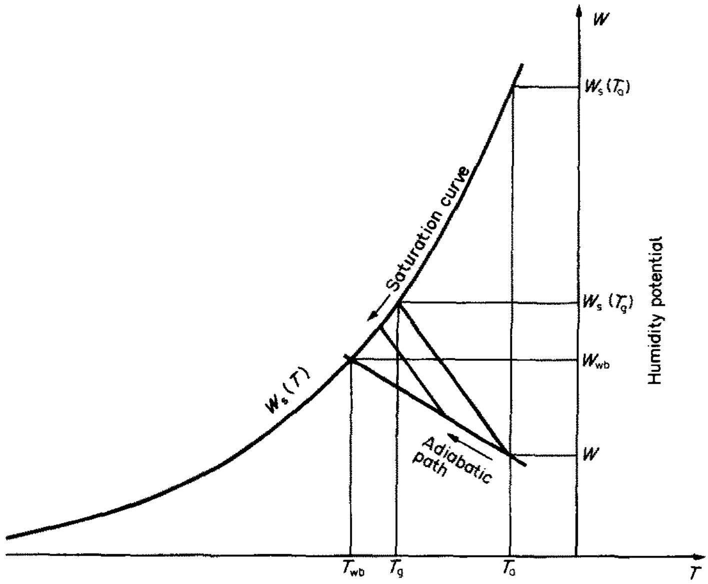

FIG. 3. Adiabatic path and humidity potential on the Grosvenor humidity chart.

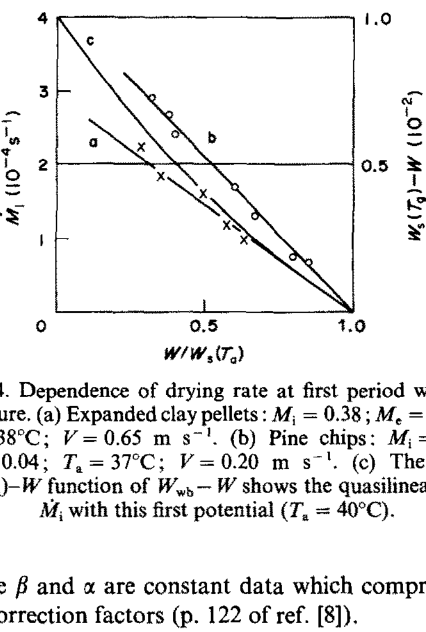

FIG. 4. Dependence of drying rate at first period with air moisture. (a) Expanded clay pellets: $M_i = 0.38$; $M_e = 0.005$; $T_a = 38°C$; $V = 0.65$ m s^{-1}. (b) Pine chips: $M_i = 1.40$; $M_e = 0.04$; $T_a = 37°C$; $V = 0.20$ m s^{-1}. (c) The curve $W_s(T_a)$–W function of $W_{wb} - W$ shows the quasilinearity of $\dot{M}_i$ with this first potential ($T_a = 40°C$).

where β and α are constant data which comprise all the correction factors (p. 122 of ref. [8]).

During all the drying periods, when air moisture and temperature vary, we need to express a hypothetic initial drying rate $\dot{M}_i$ in equation (5). To write $\dot{M}_i$ as

$$(1-\varepsilon)\rho_g\dot{M}_i = -\rho_a\beta\xi(W_s(T_g)-W) \qquad (9)$$

means that the humidity potential remains equal to $W_s(T_g) - W$ and is a T_a, W function.

For slow drying the curvature of the saturation curve $W_s(T)$ is weak and it is possible to obtain a relation of quasilinearity between $\dot{M}_i$ and several humidity potentials $W_s(T) - W$. For example the slow drying of pine chips and clay pellets shows this linear dependence as well as $W(T_{wb}) - W$ or $W_s(T_a) - W$ (Fig. 4). Thus the Van Meel hypothesis [12] of a potential humidity $W(T_{wb}) - W$ must be an admissible approximation when the air temperature and moisture levels are weak.

This form of thin layer equation can take into account mechanisms of drying and wetting near saturation of air drying. The saturated air, when subjected to cooling, either transforms itself into a mist (resulting from the presence of micro-particles of dust), or condenses on the surface of the grain remoistening it. The hypothesis of a single equation for both drying and wetting is, at best, an approximation, but is sufficient for dealing with the secondary phenomenon of wetting noticed by many authors (p. 227 of ref. [10] and ref. [6]). To put $\dot{M} = 0$ when air reaches saturation ($W = W_s(T_a)$) masks the wetting phenomenon ($\dot{M} > 0$) [13].

4. DIMENSIONLESS FORM OF THE SYSTEM

In order to simplify equations and to point out the existence of fronts, it will be convenient to express the set of equations of the model in dimensionless form. We have to establish reference parameters for the four functions M, W, T_a, T_g and two variables x, t of the drying problem of the deep bed. These parameters must be representative of physical changes during drying

$$\Delta M = M_i - M_{eo};\ M_{eo}(T_{ao}, W_o);$$

$$T_{ao} = T_a(0,t);\ W_o = W(0,t)$$

$$\Delta W = W_s(T_{as}) - W_o$$

$$\Delta T = T_{as} - T_{ao} \quad (\Delta T < 0)$$

where T_{as}, $W_s(T_{as})$ are air characteristics, results in adiabatic saturation of inlet air ($T_{as} = T_{wb}$); i.e.

$$(C_a + C_v W_o)\Delta T + L\Delta W = 0$$

$$L = L^\circ + (C_v - C_l)T_{as}. \quad (10)$$

The initial drying rate determines a reference time as

$$\tau = \frac{\Delta M}{-\dot{M}_i(W_o, T_{ao})};$$

$$\dot{M}_i(W_o, T_{ao}) = \dot{M}_{io} = -\frac{\rho_a \beta \xi}{(1-\varepsilon)\rho_g}\Delta W.$$

A reference length l will be deduced from this equation. Dimensionless functions and variables are

$$M_+ = \frac{M - M_{eo}}{\Delta M}; \quad W_+ = \frac{W - W_o}{\Delta W}; \quad T_{a+} = \frac{T_a - T_{ao}}{\Delta T};$$

$$T_{g+} = \frac{T_g - T_{ao}}{\Delta T}; \quad t_+ = \frac{t}{\tau}; \quad x_+ = \frac{x}{l}.$$

Equation (1) in dimensionless form becomes

$$\frac{l}{\tau V}\varepsilon\frac{\partial W_+}{\partial t_+} + \frac{\partial W_+}{\partial x_+} + \frac{\Delta M}{\Delta W}\frac{l}{\tau V}(1-\varepsilon)\frac{\rho_g}{\rho_a}\dot{M}_+ = 0.$$

This result defined the parameter l as

$$l = v\tau; \quad \frac{v}{V} = \frac{\rho_a}{(1-\varepsilon)\rho_g}\frac{\Delta W}{\Delta M} \quad (11)$$

and equation (1) becomes

$$\varepsilon\frac{v}{V}\frac{\partial W_+}{\partial t_+} + \frac{\partial W_+}{\partial x_+} = -\dot{M}_+. \quad (12)$$

Equation (2) expresses it in dimensionless form as

$$(1 + W_r W_+)\left(\varepsilon\frac{v}{V}\frac{\partial T_{a+}}{\partial t_+} + \frac{\partial T_{a+}}{\partial x_+}\right) = h_r(T_{g+} - T_{a+})(1 - M_{r1}\dot{M}_+) \quad (13)$$

where

$$W_r = \frac{C_v \Delta W}{C_a + W_o C_v}; \quad h_r = \frac{\xi\alpha}{\rho_a}\frac{v}{V}\frac{\tau}{C_a + W_o C_v};$$

$$M_{r1} = \frac{(1-\varepsilon)\rho_g C_v \Delta M}{\xi\alpha\tau}.$$

With the definition of τ and $\dot{M}_{io}$ the last two numbers become

$$M_{r1} = \rho_a\frac{\beta}{\alpha}C_v\Delta W; \quad h_r = \frac{\alpha}{\rho_a\beta(C_a + W_o C_v)}.$$

In order to obtain a dimensionless form of equation (4) it is convenient to rewrite it, combining it with equation (1) as

$$\left(\varepsilon\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)[\rho_a(C_a + WC_v)T_a] + (1-\varepsilon)\rho_g\frac{\partial}{\partial t}[(C_g + MC_w)T_g] = (1-\varepsilon)L^\circ\rho_g\dot{M}$$

which gives

$$\left(\varepsilon\frac{v}{V}\frac{\partial}{\partial t_+} + \frac{\partial}{\partial x_+}\right)\left[(1 + W_r W_+)\left(1 + \frac{\Delta T}{T_{ao}}T_{a+}\right)\right] + \frac{\Delta W}{\Delta M}\frac{\partial}{\partial t_+}\left[\frac{C_g + M_{eo}C_w}{C_a + W_o C_v}(1 + M_{r2}M_+)\left(1 + \frac{\Delta T}{T_{ao}}T_{g+}\right)\right] = \frac{L^\circ \Delta W}{T_{ao}(C_a + W_o C_v)}\dot{M}_+ \quad (14)$$

where

$$M_{r2} = \frac{C_w \Delta M}{C_g + M_{eo}C_w}.$$

Boundary conditions are

$$M_+(x_+, 0) = 1; \quad T_{a+}(x_+, 0) = T_{g+}(x_+, 0) = 1$$

$$M_+(0, t) = W_+(0, t) = T_{a+}(0, t) = 0.$$

Numerical integration methods can be applied to solve this set of p.d.e. [3, 13, 14]. The model requires large amounts of computer time. For slow drying performance, numerical solutions show propagation of a drying and a wetting front. We have noted the occurrence of this phenomenon for a deep bed (Fig. 1) drying with ambient air. Now we present an analysis of the equation set which allows us to show that.

5. DRYING FRONT

In the case of low temperature drying, the set of equations (12)–(14) has to be simplified in order of magnitude

$$\frac{\Delta W}{\Delta M} \ll 1; \quad \Delta W \ll 1; \quad (C_l - C_v)\Delta T \ll L^\circ. \quad (15)$$

For example in the wood chip case $\Delta M \approx 1$, $\Delta W \approx 5 \times 10^{-3}$. This leads to

$$\frac{v}{V} \ll 1 \text{ (equation (11))}.$$

Assuming an exchange only through the boundary layer around the grain leads to writing the Chilton–Colburn analogy as $h_r = 1$ thus

$$M_{r1} = \frac{C_v}{C_a + W_o C_v}\Delta W \ll 1.$$

Equations (12)–(14) are simplified as

$$\frac{\partial W_+}{\partial x_+} = -\dot{M}_+$$

$$\frac{\partial T_{a+}}{\partial x_+} = T_{g+} - T_{a+}$$

$$\frac{\partial T_{a+}}{\partial x_+} = -\dot{M}_+ \quad (16)$$

with boundary conditions

$$M_+(x_+, 0) = 1; \quad W_+(0, t_+) = T_a(0, t_+) = 0.$$

We can deduce from equations (16) that

$$T_{a+} = W_+.$$

Thus in dimensional form, using equation (10)

$$(T_a - T_{ao})(C_a + C_v W_o) + L(W - W_o) = 0. \quad (17)$$

This last equation represents adiabatic drying. We must point out the fact that this kind of drying occurs only when inequalities (15) are valid.

The last equation of the set (equations (5) and (6)) is rewritten as

$$\dot{M}_+ = \dot{M}_{+i}(W_+, T_{a+}) f(M_+)$$
$$\dot{M}_{+i} = 1 - W_+ \quad (18)$$

if we assume that the variation of the equilibrium moisture $M_e(T_a, W)$ during the drying has no notable effect on the drying rate. The set to solve becomes

$$T_{a+} = W_+$$
$$\frac{\partial T_{a+}}{\partial x_+} = T_{g+} - T_{a+}$$
$$\frac{\partial T_{a+}}{\partial x_+} = -\dot{M}_+$$
$$\dot{M}_+ = -(1 - W_+) f(M_+)$$
$$W_+(0, t_+) = 0$$
$$M_+(x_+, 0) = 1. \quad (19)$$

The substitution of W between equations $(19)_1$ and $(19)_3$ gives

$$\frac{\partial M_+}{\partial t_+} = -\left(\frac{1}{f}\frac{\partial^2 M_+}{\partial t_+ \partial x_+} - \frac{f'}{f^2}\frac{\partial M_+}{\partial t_+}\frac{\partial M_+}{\partial x_+}\right) \quad (20)$$

this equation can be integrated over t_+ as

$$M_+ = -\frac{1}{f}\frac{\partial M_+}{\partial x_+} + g(x_+).$$

The boundary conditions $M_+(x_+, 0) = 1$, $(\partial M_+/\partial x_+)(x_+, 0) = 0$ lead to $g = 1$, thus $(\partial M_+/\partial x_+) = f(M_+)(1 - M_+)$ and

$$x_+ = \int_{M_o}^{M_+} \frac{dM_+}{(1 - M_+) f(M_+)} \quad (21)$$

where $M_o = M_+(0, t)$ as M_+ varies between 1 and 0.

On the other hand, using equation $(19)_4$ at the point $x_+ = 0$

$$\frac{\partial M_o}{\partial t_+} = -f(M_o)$$

becomes

$$t_+ = -\int_1^{M_o} \frac{dM_+}{f(M_+)}. \quad (22)$$

Equations (21) and (22) define the function $M_+(x_+, t_+)$. If we said that a drying front exists, then we would have to write $M_o(t_+) = 0$ for t_+ finite value and to express a constant shape in a mobile axis.

Indeed, combining equations (21) and (22) gives

$$x_+ - t_+ = \int_1^{M_o} \frac{dM_+}{f(M_+)} + \int_{M_o}^{M_+} \frac{dM_+}{(1 - M_+) f(M_+)}.$$

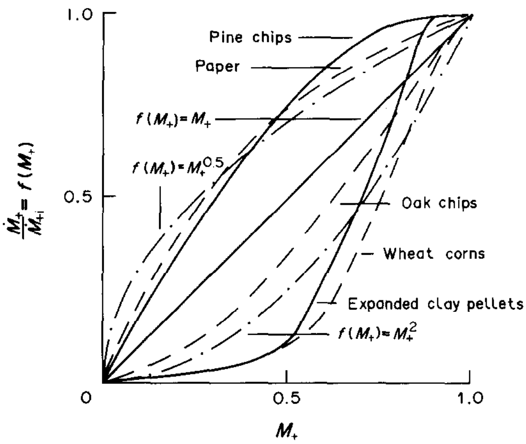

FIG. 5. Drying rate curves.

In a rearranged form we separated the term which depends in time on M_o

$$x_+ - t_+ = \underbrace{\int_1^{M_+} \frac{dM_+}{f(M_+)} + \int_0^{M_+} \frac{dM_+}{(1 - M_+)\frac{f(M_+)}{M_+}}}_{\text{Established term}}$$
$$\underbrace{-\int_0^{M_o} \frac{dM_+}{(1 - M_+)\frac{f(M_+)}{M_+}}}_{\text{Transient term}}. \quad (23)$$

When t increases, M_o and M tend toward zero and the transient term approaches zero, if the quantity to integrate remains finite. Thus $f(M_+)/M_+$ does not approach zero with M_+. Likewise another combination as $x_+ - \alpha t_+$ does not lead to a transient term.

If $f(M_+)/M_+$ does not approach zero with M_+, this latter equation gives the shape of the front $M_+(x_+ - t_+)$ and obviously the celerity v (equation (11)) which is the relationship between length and time of reference used to obtain x_+ and t_+.

For an experimental $f(M_+)$, the solution of equation (23) should be numerical but, we can apply this result to a typical rate function $f(M_+)$. In the large class of media subject to drying [10], we can select a typical curve (a) for capillary porous media such as paper, pine chips, and another (b) for hygroscopic micro-porous media such as corn and expanded clay pellets (Fig. 5). The straight line can represent glass beads, sand and oak chips. The most simple equation for these curves is $f(M_+) = M_+^n$, with $n < 1$ for (a) and $n > 1$ for (b). Then solution of equation (22) is

$$M_o = [1 - (1 - n)t_+]^{1/(1-n)} \quad (24)$$

thus

$$n < 1\,;\; M_o\left(\frac{1}{1-n}\right) = 0$$

$$n > 1\,;\; M_o \to 0 \text{ when } t \to \infty \text{ and}$$

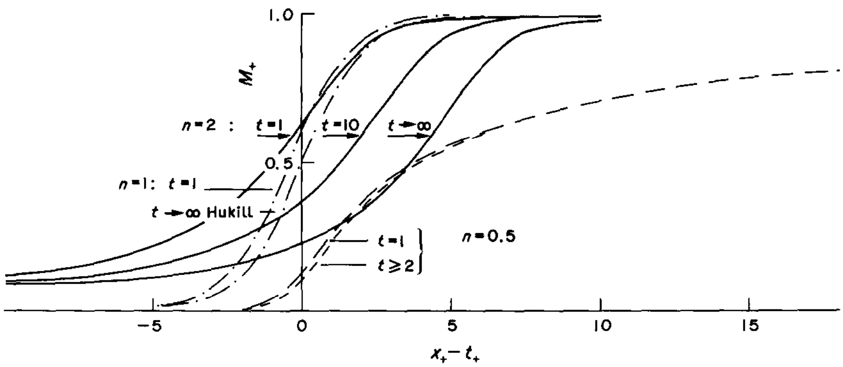

FIG. 6. Theoretical shape of drying front.

$$\frac{f(M_+)}{M_+} \to 0 \text{ when } M_+ \to 0.$$

For this last case, although the drying front does not exist strictly speaking, one can define a quasi drying front, for example as

$$M_o = 10^{-2} \to t_{+1\%} = \frac{1}{n-1}(100^{n-1}-1).$$

As an illustration, let $n = 1/2$ and 2. With the result of equation (24), integration of equation (23) gives

$$\begin{cases} n = \frac{1}{2}; \quad M_o = \left(1-\frac{t_+}{2}\right)^2, \quad t \leqslant 2; \quad M_o = 0, \quad t \geqslant 2 \\ x_+ - t_+ = -2 + \ln\frac{(1+\sqrt{M_+})(1-\sqrt{M_o})}{(1-\sqrt{M_+})(1+\sqrt{M_o})} + 2\sqrt{M_o} \end{cases}$$

$$n = 2; \quad M_o = \frac{1}{t_+ + 1};$$

$$x_+ - t_+ = 1 - \frac{1}{M_+} + \ln\frac{M_+}{1-M_+} + \ln t_+.$$

The celerity of the pseudo front is $C_+ = (\mathrm{d}x_+/\mathrm{d}t_+) = 1 + 1/t_+$, or in dimensional form $C = v(1+\tau/t)$.

The particular case of $n = 1$ is deduced from equations (22) and (23)

$$M_o = \mathrm{e}^{-t_+}; \quad x_+ - t_+ = \ln\left(\frac{M_+}{1-M_+}(1-\mathrm{e}^{-t_+})\right)$$

or in explicit form for M_+

$$M_+ = \frac{1}{1+\mathrm{e}^{-x_+}(\mathrm{e}^{t_+}-1)}. \tag{25}$$

Hukill [7] obtains this last expression with particular boundary conditions; Schlunder [15] and Keey [8] give it as an example of batch drying. Ashworth [16] obtains a solution for $n = 1/2$.

Figure 6 compares the shape of these theoretical fronts and shows that in the transient phase the front is distorted very little.

This analysis shows that the logarithmic model is a particular drying model which allows a double linearity for the thin layer equation $\dot{M}$ with $M-M_e$ (characteristic of the product) and $\dot{M}_i$ with $W_s(T_{wb}) - W$ (characteristic of slow drying). Without this last property, the shape of the drying front would be somewhat different (Fig. 7).

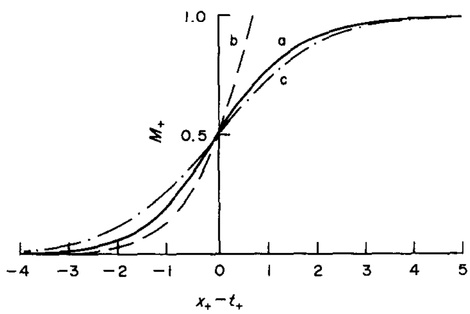

FIG. 7. Shape of drying front (calculations): (a) pine chips with $W_s(T_a)$–W factor; (b) pine chips without $W_s(T_a)$–W factor; (c) solution of Hukill.

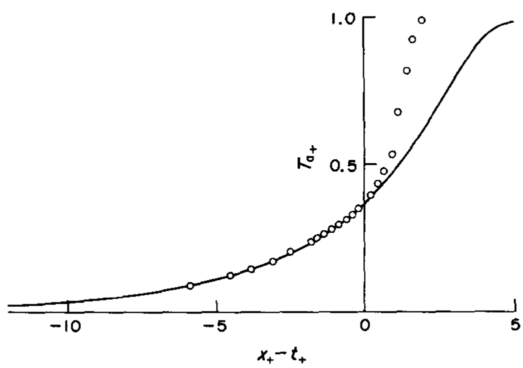

FIG. 8. Comparison of shape of drying front obtained by experiments (○) and calculations (——). Case of expanded clay pellets: $M_i = 0.42$; $M_e = 0.01$; $T_{ao} = 30°C$; $T_{as} = 17.5°C$; $V = 0.67\ \mathrm{m\,s^{-1}}$; $W_o/W_s(T_{ao}) = 0.30$. v drying: measured (18.9–$19.4 \times 10^{-6}\ \mathrm{m\,s^{-1}}$); calculated ($18.2 \times 10^{-6}\ \mathrm{m\,s^{-1}}$).

Experiments on expanded clay pellets validate the model (Fig. 1). The measure of celerity of the drying front confirms the calculation with some percentage of difference (5%). The agreement on the drying shape is good when the measure is reliable (Fig. 8). As the drying front comes upon the thermocouple the air temperature gradient is high between the grain (the

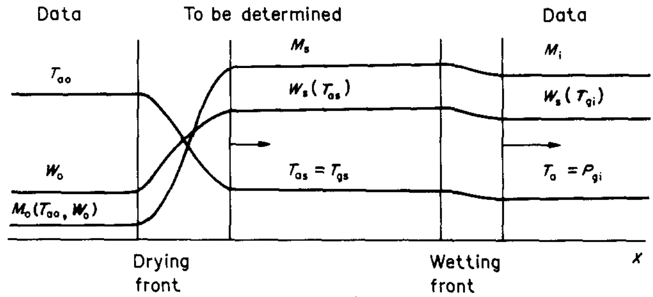

FIG. 9. Boundary conditions.

surface is saturated) and the probe indication is suspect.

6. WETTING OR DRYERATION FRONT

A second type of front occurs at the start of the drying process. Inlet air on the drier becomes suddenly colder and saturated, then crosses the product generally at a lower temperature and thus condenses on surfaces. Since this front is fugacious (celerity much more important than drying front) and its temperature variation is weak (some 0.1°C), this phenomenon cannot be detected.

In the dryeration process (p. 176 of ref. [3]) the cereal is removed from the drier at a moisture content of about 2% above the desired value. The heat of the hot grain increases the drying power of the ventilation air the temperature of which can increase after condensation.

The dimensionless equations can be obtained in the same way as previously but with new reference parameters in relation to the new physical problem (Fig. 9)

$$\Delta M = M_i - M_s; \quad M_s(T_{as}, W_s(T_{as})).$$

The boundary condition M_s will be determined later; T_{as} given equation (10)

$$\Delta W = W_s(T_{gi}) - W_s(T_{as}); \quad \Delta T = T_{gi} - T_{as}$$

$$M_+ = \frac{M - M_s}{\Delta M}; \quad W_+ = \frac{W_s(T_a) - W_s(T_{as})}{\Delta W}$$

$$T_+ = \frac{T_a - T_{as}}{\Delta T}; \quad \tau = -\frac{\Delta M}{\dot{M}_i}$$

$$\dot{M}_i = -\frac{\rho_a \xi \beta}{\rho_g(1-\varepsilon)}(W_s(T_{gi}) - W_s(T_{as}))$$

$$\# -\frac{\rho_a \xi \beta}{\rho_g(1-\varepsilon)} W_s'(T_{as})\Delta T.$$

The dimensionless system has the same form as previously (equations (12)–(15)). Now, the simplifications allowed in this case are

$$\Delta W \approx \Delta M \ll 1; \quad \frac{v}{V} = \frac{\rho_a}{(1-\varepsilon)\rho_g}\frac{\Delta W}{\Delta M} \ll 1;$$

$$(C_l - C_v)\Delta T \ll L^\circ$$

this leads to

$$W_r \approx \Delta W \ll 1; \quad M_{r1} \approx \Delta W \ll 1; \quad M_{r2} \approx \Delta M \ll 1.$$

The system becomes

$$\frac{\partial W_+}{\partial x_+} = -\dot{M}_+ \tag{26}$$

$$\frac{\partial T_{a+}}{\partial x_+} = T_{g+} - T_{a+} \tag{27}$$

$$\frac{\partial T_{a+}}{\partial x_+} + \frac{C_g + M_i C_w}{C_a + W_s C_v}\frac{\Delta W}{\Delta M}\dot{T}_{g+} = \frac{L^\circ \Delta W}{(C_a + W_s C_v)\Delta T}\dot{M}_+ \tag{28}$$

$$\dot{M}_+ = \frac{W_s'(T_a)}{W_s'(T_{as})}(T_{g+} - T_{a+}). \tag{29}$$

An additional assumption is valid

$$W_s'(T_a) \# W_s'(T_{as}).$$

From this, taking into account equations (27) and (29), we can express equation (28) as

$$(C_g + M_i C_w)\Delta W \dot{T}_{g+} = \left(L\frac{\Delta W}{\Delta T} + C_a + W_s C_v\right)\Delta M \dot{M}_+$$

the integration of which between $t_+ = 0$ ($T_{g+} = M_+ = 1$) and $t_+ \to \infty$ ($T_{g+} = M_+ = 0$) gives

$$(C_g + M_i C_w)\Delta T \Delta W = (L\Delta W + (C_a + W_s C_v)\Delta T)\Delta M. \tag{30}$$

Since ΔT, ΔW are known, equation (30) determines ΔM, thus M_s. Therefore, celerities of drying and wetting fronts are deduced from equation (11), with respective values of ΔW, ΔM. Since the temperature variation is very weak, we do not give the shape of this front. For example in the case of expanded clay pellets the result of the calculation is (data Fig. 8)

$$v_{\text{drying}} = 1.95 \times 10^{-5}\,\text{m s}^{-1}$$

$$v_{\text{wetting}} = 1.8 \times 10^{-3}\,\text{m s}^{-1}; \quad \Delta T_{\text{wetting}} = 0.2°\text{C}.$$

7. CONCLUSION

For drying at low temperature, the Van Meel assumption of a potential humidity calculated from adiabatic saturation is valid. The slow continual drying in a thick bed without recycling can have an analytical solution which shows the removal of a drying front with a constant celerity. The form of the drying rate curve determines the shape of the front and the importance of the transient effect. The transformation of the air is adiabatic. At the beginning of the drying process a fugacious wetting front occurs, the celerity of which can be calculated with boundary conditions.

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APPENDIX. PARTIAL DIFFERENTIAL EQUATION MODEL

Rather than writing a mass and energy balance in an elementary volume, it is easier to use a general balance equation for a point of porous medium of static bed. Taking into account previously indicated assumptions, let $f(x, t)$ be a physical parameter dispersed in air and grain. One can note

$$f = \varepsilon f_a + (1-\varepsilon) f_g.$$

Balance of f in a volume v is

$$\frac{d}{dt}\iiint_v [\varepsilon f_a + (1-\varepsilon) f_g]\, dv = \iiint_v \phi\, dv \qquad \text{(A1)}$$

where ϕ is the production density of f (d/dt is substantial derivation).

Development and identification of the two members of equation (1), give

$$\varepsilon \frac{\partial f_a}{\partial t} + \operatorname{div} f_a \mathbf{V} + (1-\varepsilon)\frac{\partial f_g}{\partial t} = \phi. \qquad \text{(A2)}$$

For low temperature drying we can assume a uniform flow rate velocity and equation (2) becomes

$$\varepsilon \frac{\partial f_a}{\partial t} + V\frac{\partial f_a}{\partial x} + (1-\varepsilon)\frac{\partial f_g}{\partial t} = \phi. \qquad \text{(A3)}$$

The mass balance equation for wet air is obtained by putting $f = \varepsilon\rho_a W + (1-\varepsilon)\rho_g M$; $\phi = 0$, as

$$\rho_a\left(\varepsilon\frac{\partial W}{\partial t} + V\frac{\partial W}{\partial x}\right) + (1-\varepsilon)\rho_g\frac{\partial M}{\partial t} = 0. \qquad \text{(A4)}$$

The enthalpy balance equation for wet air is obtained by putting

$$f = \varepsilon\rho_a[(C_a + WC_v)T_a + L^\circ W]$$

$$\phi = \xi\alpha(T_g - T_a) - (1-\varepsilon)\rho_g(C_v T_g + L^\circ)\frac{\partial M}{\partial t}$$

where ρ_a is the volumetric mass of dry air (constant); using equations (3) and (4) the result is

$$\rho_a(C_a + WC_v)\left(\varepsilon\frac{\partial T_a}{\partial t} + V\frac{\partial T_a}{\partial x}\right) = \xi\alpha(T_g - T_a) - (1-\varepsilon)\rho_g C_v(T_g - T_a)\frac{\partial M}{\partial t}. \qquad \text{(A5)}$$

The enthalpy balance of air and grain together is obtained by putting

$$f = \varepsilon\rho_a(C_a T_a + W(C_v T_a + L^\circ)) + (1-\varepsilon)\rho_g(C_g + MC_w)T_g$$

$$\phi = 0$$

using equations (3) and (4), the result is

$$\rho_a(C_a + C_v W)\left(\varepsilon\frac{\partial T_a}{\partial t} + V\frac{\partial T_a}{\partial x}\right) + (1-\varepsilon)\rho_g(C_g + MC_w)\frac{\partial T_g}{\partial t} = L^*(1-\varepsilon)\rho_g\frac{\partial M}{\partial t} \qquad \text{(A6)}$$

where

$$L^* = L^\circ + C_v T_a - C_w T_g.$$

The enthalpy balance of grain is obtained by finding the difference between equations (6) and (5) as

$$(1-\varepsilon)\rho_g(C_g + MC_w)\frac{\partial T_g}{\partial t} = \xi\alpha(T_a - T_g) + L\rho_g(1-\varepsilon)\frac{\partial M}{\partial t} \qquad \text{(A7)}$$

where

$$L = L^\circ + (C_v - C_w)T_g.$$

SIMULATION DU SECHAGE LENT EN COUCHE EPAISSE DE PRODUITS GRANULAIRES

Résumé—L'équation de cinétique utilisée dans les modèles de simulation de séchage à basse température qui explicite l'humidité de l'air sous forme d'un potentiel d'humidité calculé par rapport à la saturation adiabatique est légitime. Nous proposons une transformation des équations aux dérivées partielles du modèle en une forme adimensionnelle dont les grandeurs de référence sont reliées au problème physique. Le séchage lent permet un certain nombre de simplifications qui peuvent conduire à une solution analytique dépendant de l'équation de cinétique de séchage. On montre comment obtenir les caractéristiques des fronts de séchage et de réhumidification.

SIMULATION DER LANGZEITTROCKNUNG IN DICKSCHICHTIGEN GRANULATEN

Zusammenfassung—Die Gleichung für die Trocknungsgeschwindigkeit, die in Simulationsmodellen für die Niedertemperatur-Trocknung verwendet wird, stellte sich als gültig heraus. Diese Gleichung berücksichtigt den Einfluß der Luftfeuchtigkeit als Feuchtepotential, welches aus der adiabaten Sättigung berechnet wird. Es wird vorgeschlagen, das partielle Differentialgleichungsmodell in ein System dimensionsloser Kennzahlen umzuformen, wobei die Referenz-Parameter in Übereinstimmung mit den physikalischen Gegebenheiten stehen müssen. Da der Trocknungsvorgang langsam abläuft, sind einige Vereinfachungen möglich. Dies führt zu einer analytischen Lösung, die von der Trocknungsgeschwindigkeit abhängt. Weiterhin wird gezeigt, wie das Verhalten von Trocken- und Benetzungsfront ermittelt werden kann.

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА МЕДЛЕННОЙ СУШКИ ТОЛСТЫХ СЛОЕВ ГРАНУЛИРОВАННЫХ МАТЕРИАЛОВ

Аннотация—Показана применимость уравнения скорости сушки в качестве расчетной модели для описания процесса низкотемпературной сушки, при которой влажность воздуха является потенциалом сушки, рассчитываемым по адиабатическому насыщению. Модель, содержащая уравнения в частных производных, преобразована в безразмерную, в которой исходные параметры согласуются с физической моделью. Медленная сушка позволяет принять ряд упрощений и получить аналитическое решение, определяемое скоростью сушки. Получены характеристики фронтов сушки или увлажнения.